Fuzzy Compromise Approach for
Solving Interval-Valued Fractional Multi-Objective
Multi-Product Solid Transportation Problems

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Abstract. In this paper, a fractional multi-objective multi-product solid transportation problem with interval costs, supply, demand, and conveyances is investigated based on fuzzy programming approach. To minimize the problem, the order relations that represent the decision maker's (DM) performance between interval costs, supply, demand and conveyances are defined by the right limit, and the left limit. Through the deterministic problem is obtained, a fuzzy programming approach is applied by defining membership functions. A linear membership function is being used for obtaining optimal compromise solution. Finally, a numerical example is given to the utility of the approach.

Keywords: Multi-Objective Multi-Product Solid Transportation Problem; Interval Numbers; Inexact Programming; linear Membership Function; Fuzzy Programming; Fractional Programming.
1. Introduction
membership function that's by considering deviations of objective function from the goal value to obtain the target goal for a multiobjective TP. Fractional Programming (FP) is a decision problem arises to optimize the ratio subject to constraints. In real world decision situations, MOLFP programming arises very frequently. As, for instance, the ratio between inventory& sales, actual cost & standard cost, output& employee, measuring relative efficiency of decision making unit in the public/ or nonprofit sectors, Data Envelopment Analysis (DEA) & many other areas of economics, non- economics and indirect applications. Charnes and Cooper (1962) studied a linear fractional programming (LFP) problem and showed that it can be optimized by solving two linear programs. Ammar and Khalifa (2009) studied LFP problem with fuzzy parameters. Ammar and Khalifa (2004) introduced a parametric approach for solving multi-criteria LFP problem. Luhandjula (1984) applied fuzzy programming approach for solving MOLFP problem. Nykowski and Zolkiewski (1985) solved the MOLFP problem by converting it into the multi- objective linear programming (MOLP) problem. Gupta and Chakraborty (1999) have been introduced a methodology for a restricted class of MOLFP problem in the sense that there exists some values of decision variables for which the numerator is positive and the denominator is positive for all values of decision variables in the feasible region, and then applied fuzzy approach for solving the problem by defining a linear membership function. Radhakrishnan and Anukokila (2014) used fractional goal programming approach for solving STP with interval cost.

The rest of the paper is organized as : In section 2; some preliminaries need are presented. In section 3, a multi- objective multi- product solid transportation problem with interval costs, supply, demand is formulated. In section 4, a fuzzy programming approach for solving the problem is given. In section 5, an interactive procedure for obtaining the optimal compromise solution is suggested. In section 6, A numerical example is given for illustration. Finally, some concluding remarks are reported in section 7.
2. Preliminaries
In order to discuss our problem conveniently, we shall state some
necessary results on interval arithmetic (see, Moore, 1979; Kauffmann
and Gupta, 1988).

Let \( I(R) = \{ [a^L, a^U] : a^L, a^U \in R = (-\infty, \infty), a^L \leq a^U \} \) denote the set of all
closed interval numbers on \( R \), where \( a^L \) is the left limit and \( a^U \) is the
upper the right limit.

**Definition 1.** Assume that: \( A = [a^L, a^U], B = [b^L, b^U] \in I(R) \), we define:

(i) \[ [a^L, a^U] (+) [b^L, b^U] = [a^L + b^L, a^U + b^U] \] \hspace{1cm} (1)

(ii) \[ [a^L, a^U] (-) [b^L, b^U] = [a^L - b^U, a^U - b^L] \] \hspace{1cm} (2)

(iii) \[ [a^L, a^U] (-) [b^L, b^U] = \left[ \min (a^L b^L, a^L b^U, a^U b^L, a^U b^U), \max (a^L b^L, a^L b^U, a^U b^L, a^U b^U) \right] \] \hspace{1cm} (3)

(iv) \[ [a^L, a^U] (/) [b^L, b^U] = \left[ \min \left( \frac{a^L}{b^L}, \frac{a^L}{b^U}, \frac{a^U}{b^L}, \frac{a^U}{b^U} \right), \max \left( \frac{a^L}{b^L}, \frac{a^L}{b^U}, \frac{a^U}{b^L}, \frac{a^U}{b^U} \right) \right] \] \hspace{1cm} (4)

(v) \[ k[a^L, a^U] = \begin{cases} [ka^L, ka^U], k \geq 0 \\ [ka^U, ka^L], k < 0 \end{cases} \] \hspace{1cm} (5)

Where, \( k \in R \).

(vi) The order relation \( \leq LU \) in \( I(R) \) is defined by:

\[ [a^L, a^U] (\leq LU) [b^L, b^U] \] if and only if \( a^L \leq b^L, a^U \leq b^U \), \hspace{1cm} (6.1)

\[ [a^L, a^U] (\leq LU) [b^L, b^U] \] if and only if \[ [a^L, a^U] (\leq LU) [b^L, b^U], \text{ and } [a^L, a^U] (\neq) [b^L, b^U]. \] \hspace{1cm} (6.2)

**Proposition 1.** (Ishibuchi and Tanaka (1990)).
If \( A = [a^L, a^U] (\leq LU) B = [b^L, b^U] \), then \( A = B \).

**Proposition 2.** (Ishibuchi and Tanaka (1990)).
\( A = B \) if and only if, (6.1), and (6.2) hold.

**Proposition 3.** (Gupta and Chakraborty, 1999).
If \( d^T > 0 \), and \( d_0 > 0 \), then
\[ N = \frac{c^T x + c_0}{d^T x + d_0}, x \geq 0 \] has the maximum value \( \bar{N} = \max \{ c_i/d_i, c_0/d_0, i = 1,2,\ldots,n \} \) and the minimum value \( \underline{N} = \min \{ c_i/d_i, c_0/d_0, i = 1,2,\ldots,n \} \).

3. Problem formulation and solution concepts

A fractional multi-objective multi-product solid transportation problem is formulated as follows

\[
\min Z_r = \frac{N_r(x)}{M_r(x)} = \frac{\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \begin{bmatrix} c_{ijk}^p \end{bmatrix}^L \begin{bmatrix} c_{ijk}^p \end{bmatrix}^U x_{ijk}^p}{\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \begin{bmatrix} d_{ijk}^p \end{bmatrix}^L \begin{bmatrix} d_{ijk}^p \end{bmatrix}^U x_{ijk}^p},
\]

subject to

\[
\begin{align*}
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p & \leq \begin{bmatrix} (a_i^p) \end{bmatrix}^L \begin{bmatrix} (a_i^p) \end{bmatrix}^U = A_i; \forall i, p, \\
\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p & \geq \begin{bmatrix} (b_j^p) \end{bmatrix}^L \begin{bmatrix} (b_j^p) \end{bmatrix}^U = B_j; \forall j, p, \\
\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p & \leq \begin{bmatrix} (e_k) \end{bmatrix}^L \begin{bmatrix} (e_k) \end{bmatrix}^U = C_k; \forall k,
\end{align*}
\]

\[ x_{ijk}^p \geq 0; \forall i, j, k, p. \quad (7) \]

Where \( p(=1,2,\ldots,l) \) products can be transported from \( m \) origins \( A_i(i = 1,2,\ldots,m) \) to \( n \) destination \( B_j(j = 1,2,\ldots,n) \) by means of \( C_k(k = 1,2,\ldots,K) \) conveyances, and \( r(=1,2,\ldots,S) \) objectives are to be minimized.

To solve the problem (7), the following conditions must be satisfied:

\[
\sum_{i=1}^{m} \begin{bmatrix} (a_i^p) \end{bmatrix}^L \begin{bmatrix} (a_i^p) \end{bmatrix}^U \geq \sum_{j=1}^{n} \begin{bmatrix} (b_j^p) \end{bmatrix}^L \begin{bmatrix} (b_j^p) \end{bmatrix}^U, p = 1,2,\ldots,l
\]

and

\[
\sum_{k=1}^{K} \begin{bmatrix} (e_k) \end{bmatrix}^L \begin{bmatrix} (e_k) \end{bmatrix}^U \geq \sum_{p=1}^{l} \sum_{j=1}^{n} \begin{bmatrix} (b_j^p) \end{bmatrix}^L \begin{bmatrix} (b_j^p) \end{bmatrix}^U.
\]

**Definition 2.** (Interval-valued efficient solution). A point \( x^* \in X(A_i, B_j, E_k), i = 1,2,\ldots,m; j = 1,2,\ldots,n; k = 1,2,\ldots,K, \) is said to be interval-valued efficient solution to the problem (7) if and only if there
does not exist another \( x \in X(A_i, B_j, E_k) \), such that: \( Z_r(x) \leq Z_r(x^*) \), and 
\( Z_r(x) < Z_r(x^*) \) for at least one \( r \).

It follows that the problem (7) can be rewritten as follows

\[
\text{Min}(Z_r)^U = \left( \frac{N_r(x)}{(M_r(x))^L} \right)^U = \frac{\sum_{p=1}^{s} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (c_{ijk}^{rp})^U x_{ijk}^p}{\sum_{p=1}^{s} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (d_{ijk}^{rp})^L x_{ijk}^p}, \quad r = 1, 2, ..., S
\]

Subject to

\[
\begin{align*}
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p & \leq (a_i^p)^U; \forall i, p, \\
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p & \geq (a_i^p)^L; \forall i, p, \\
\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p & \leq (b_j^p)^U; \forall j, p, \\
\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p & \geq (b_j^p)^L; \forall j, p, \\
\sum_{p=1}^{s} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p & \leq (e_k)^U, \forall k, \\
\sum_{p=1}^{s} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p & \geq (e_k)^L; \forall k, \\
x_{ijk}^{rp} & \geq 0; \forall i, j, k, p.
\end{align*}
\]

4. Fuzzy programming approach for solving the problem

Bellman and Zadeh (1970) introduced three basic concepts: fuzzy goal \((G)\), fuzzy constraints \((T)\), and fuzzy decision \((D)\) and explored the applications of these concepts to the decision making under fuzziness. Their fuzzy decision is defined as follows:

\[
D = G \cap T
\]

This problem is characterized by the membership function

\[
\mu_D(x) = \min(\mu_G(x), \mu_T(x)),
\]

To define the membership function of the problem (8), let us follow:
Calculate the individual minimum as:
\[
((Z_r)^u)(Z_r)^u_{\text{min}} = \min \{ (Z_r)^u(x) : x \in X \},
\]
(11)
The individual maximum as:
\[
((Z_r)^u)(Z_r)^u_{\text{max}} = \max \{ (Z_r)^u(x) : x \in X \},
\]
(12)
Where, \(X\) is the feasible region of the problem (8).

On the basis of definition of \((Z_r)^u_{\text{min}}\), and \((Z_r)^u_{\text{max}}\), the membership functions for the problem (8) as follows (Biswal (1992)):
\[
\mu_r((Z_r(x))^u) = \begin{cases} 
1, & \text{if } (Z_r(x))^u \leq (Z_r)^u_{\text{min}}, \\
\frac{(Z_r)^u_{\text{max}} - (Z_r(x))^u}{(Z_r)^u_{\text{max}} - (Z_r)^u_{\text{min}}}, & \text{if } (Z_r)^u_{\text{min}} < (Z_r(x))^u < (Z_r)^u_{\text{max}}, \\
0, & \text{if } (Z_r(x))^u \geq (Z_r)^u_{\text{max}}, 
\end{cases} 
\]
(13)
Following the fuzzy decision of Bellman and Zadeh (1970) with the linear membership function (13) a fuzzy programming model to the problem (8) can be written as follows:
\[
\max \min_{r=1,2,...,s} \{ \mu_r((Z_r)^u) \}, 
\]
Subject to
\[
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p \leq (a_i^p)^u, \quad i = 1,2,\ldots,m; \quad p = 1,2,\ldots,l,
\]
\[
\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p \geq (a_i^p)^L, \quad i = 1,2,\ldots,m; \quad p = 1,2,\ldots,l,
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p \leq (b_j^p)^u, \quad j = 1,2,\ldots,n; \quad p = 1,2,\ldots,l,
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p \geq (b_j^p)^L, \quad j = 1,2,\ldots,n; \quad p = 1,2,\ldots,l,
\]
\[
\sum_{p=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{K} x_{ijk}^p \leq (e_k)^u, \quad k = 1,2,\ldots,K,
\]
\[
\sum_{p=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{K} x_{ijk}^p \geq (e_k)^L, \quad k = 1,2,\ldots,K,
\]
Problem (14) can be transformed into the following problem

$$\max_{x} \lambda$$

Subject to

$$\mu_r((Z_r(x))^U) \geq \lambda,$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p \leq (a_i^p)^U, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^p \geq (a_i^p)^L, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p \leq (b_j^p)^U, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^p \geq (b_j^p)^L, j = 1,2,\ldots,n; p = 1,2,\ldots,l,$$

$$\sum_{p=1}^{n} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p \leq (e_k)^U, k = 1,2,\ldots,K,$$

$$\sum_{p=1}^{n} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^p \geq (e_k)^L, k = 1,2,\ldots,K,$$

$$x_{ijk}^r \geq 0; \forall i,j,k,p, 0 < \lambda \leq 1.$$

Where, $\lambda$ is an auxiliary variable.

By the transformation $= xt$, the problem (15) becomes

$$\max_{x} \lambda$$

Subject to

$$\mu_r((N_r(y/t))^U) \geq \lambda,$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} (y/t)^p_{ijk} \leq (a_i^p)^U, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} (y/t)^p_{ijk} \geq (a_i^p)^L, i = 1,2,\ldots,m; p = 1,2,\ldots,l,$$
\[
\sum_{i=1}^{m} \sum_{k=1}^{K} (y/t)^p_{ijk} \leq (b^p_j)^U, j = 1,2,\ldots, n; p = 1,2,\ldots, l,
\]
\[
\sum_{i=1}^{l} \sum_{k=1}^{m} \sum_{j=1}^{n} (y/t)^p_{ijk} \geq (b^p_j)^L, j = 1,2,\ldots, n; p = 1,2,\ldots, l,
\]
\[
\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} (y/t)^p_{ijk} \leq (e_k^p)^U, k = 1,2,\ldots, K,
\]
\[
\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} (y/t)^p_{ijk} \geq (e_k^p)^L, k = 1,2,\ldots, K,
\]
\[
y^p_{ijk} \geq 0, t > 0; \forall i,j,k,p; 0 < \lambda \leq 1. \tag{16}
\]

5. A solution procedure

**Step 1:** Calculate the individual minimum and maximum of each objective function under the given constraints.

**Step 2:** Define the membership function, \( \mu_r ((Z_r(x))^U) \), and \( r = 1,2,\ldots, S \), as mentioned in equation (13).

**Step 3:** Construct the fuzzy programming problem (14), and its equivalent linear programming problem (15).

**Step 4:** Solve problem (16) by using integer-programming approach to obtain an integer optimal compromise solution and hence evaluate the \( S \) objective functions at the resulted optimal compromise solution.

**Step 5:** Stop.

6. Numerical example

Consider the following problem with \( p = 1,2 = i = k, j = 1,2,3 \)

\[
\text{Min} Z_1 = \frac{\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left( (c^{1p}_{ijk})^L, (c^{1p}_{ijk})^U \right) x^p_{ijk}}{\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left( (d^{1p}_{ijk})^L, (d^{1p}_{ijk})^U \right) x^p_{ijk}}
\]
\[ \text{Min} \mathbf{Z}_2 = \frac{\sum_{p=1}^{2} \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{2} \left[ \left( c_{ij}^{2p} \right)^L, \left( c_{ij}^{2p} \right)^U \right] x_{ijk}^p}{\sum_{p=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} \left[ \left( d_{ij}^{2p} \right)^L, \left( d_{ij}^{2p} \right)^U \right] x_{ijk}^p}, \]

Subject to

\[ \sum_{j=1}^{3} \sum_{k=1}^{2} x_{1jk}^1 \leq [(a_1^1)^L, (a_1^1)^U] = [24, 26] \]
\[ \sum_{j=1}^{3} \sum_{k=1}^{2} x_{1jk}^2 \leq [(a_1^2)^L, (a_1^2)^U] = [32, 35] \]
\[ \sum_{j=1}^{3} \sum_{k=1}^{2} x_{2jk}^1 \leq [(a_2^1)^L, (a_2^1)^U] = [34, 37] \]
\[ \sum_{j=1}^{3} \sum_{k=1}^{2} x_{2jk}^2 \leq [(a_2^2)^L, (a_2^2)^U] = [28, 30] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{1lk}^1 \geq [(b_1^1)^L, (b_1^1)^U] = [16, 19] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{1lk}^2 \geq [(b_1^2)^L, (b_1^2)^U] = [23, 25] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{2lk}^1 \geq [(b_2^1)^L, (b_2^1)^U] = [20, 22] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{2lk}^2 \geq [(b_2^2)^L, (b_2^2)^U] = [18, 19] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{3lk}^1 \geq [(b_3^1)^L, (b_3^1)^U] = [15, 18] \]
\[ \sum_{l=1}^{2} \sum_{k=1}^{2} x_{3lk}^2 \geq [(b_3^2)^L, (b_3^2)^U] = [17, 19] \]
\[ \sum_{l=1}^{2} \sum_{m=1}^{2} \sum_{n=1}^{2} x_{ijk}^p \leq [(e_1)^L, (e_2)^U] = [48, 51] \]
\[ \sum_{p=1}^{2} \sum_{l=1}^{2} \sum_{j=1}^{2} x_{ijk}^p \leq [(e_2)^L, (e_2)^U] = [53, 56] \]

\[ x_{ijk}^p \geq 0; \forall i, j, k, p. \]

Where, the unit transportation penalties are given in Tables 1-8 as follow:
Table 1. Penalties/costs $c_{ijk}^{11}$

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Table 2. Penalties/costs $d_{ijk}^{11}$

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Table 3. Penalties/costs $c_{ijk}^{12}$

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Table 4. Penalties/ costs $d_{ijk}^{12}$

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Table 5. Penalties/ costs $c_{ijk}^{21}$

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Table 6. Penalties/ costs $d_{ijk}^{21}$

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<td>2</td>
<td>10,14</td>
<td>7,9</td>
</tr>
</tbody>
</table>

Table 7. Penalties/ costs $c_{ijk}^{22}$

<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7,9</td>
<td>6,8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9,13</td>
<td>6,8</td>
</tr>
</tbody>
</table>

Table 8. Penalties/ costs $d_{ijk}^{22}$

<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1,3</td>
<td>2,6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>10,12</td>
<td>13,15</td>
</tr>
</tbody>
</table>
Referring to the problem (8), the equivalent multi-objective ordinary problem is

\[ Min(Z_1)^U = \frac{\sum_{p=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} (c_{ijk}^p)^U}{\sum_{p=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} (d_{ijk}^p)^L} x_{ijk}^p, \]

\[ Min(Z_2)^U = \frac{\sum_{p=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} (c_{ijk}^p)^L}{\sum_{p=1}^{2} \sum_{i=1}^{3} \sum_{j=1}^{2} \sum_{k=1}^{3} (d_{ijk}^p)^L} x_{ijk}^p, \]

Subject to

\[ x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{112}^1 + x_{122}^1 + x_{132}^1 \leq 26, \]
\[ x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 \leq 35, \]
\[ x_{111}^2 + x_{121}^2 + x_{131}^2 + x_{112}^2 + x_{122}^2 + x_{132}^2 \leq 37, \]
\[ x_{211}^2 + x_{221}^2 + x_{231}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \leq 30, \]
\[ x_{111}^1 + x_{121}^1 + x_{112}^1 + x_{122}^1 \geq 16, \]
\[ x_{211}^2 + x_{221}^2 + x_{212}^2 \geq 23, \]
\[ x_{121}^1 + x_{122}^1 + x_{122}^2 \geq 20, \]
\[ x_{121}^1 + x_{221}^1 + x_{122}^1 + x_{222}^1 \geq 18, \]
\[ x_{131}^1 + x_{231}^1 + x_{132}^1 + x_{232}^1 \geq 15, \]
\[ x_{231}^1 + x_{231}^2 + x_{132}^2 + x_{232}^2 \geq 17, \]
\[ x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^2 + x_{231}^1 + x_{112}^1 + x_{212}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \leq 51, \]
\[ x_{112}^1 + x_{212}^1 + x_{122}^1 + x_{132}^1 + x_{222}^1 + x_{112}^2 + x_{212}^2 + x_{122}^2 + x_{222}^2 + x_{132}^2 + x_{232}^2 \leq 56, \]
\[ x_{ijk}^p \geq 0; \forall i, j, k, p. \]

Where,

\[
(c_{ijk}^p)^U = \begin{pmatrix}
9x_{111}^1 + 10x_{121}^1 + 14x_{131}^1 + 12x_{211}^1 + 9x_{212}^1 + 15x_{231}^1 + 13x_{112}^1 + 10x_{122}^1 + 12x_{132}^1 + 13x_{212}^2 + 10x_{222}^1 + 18x_{232}^1 + 12x_{112}^2 + 12x_{122}^1 + 14x_{132}^1 + 14x_{212}^2 + 12x_{232}^1 + 12x_{222}^2 + 15x_{232}^2
\end{pmatrix}
\]
\[ \left( d_{ijk}^{1p} \right)^L = \left( \begin{array}{c} 2x_{111}^1 + 1x_{121}^1 + 4x_{131}^1 + 3x_{211}^1 + 7x_{221}^1 + 11x_{231}^1 + 5x_{112}^1 + 4x_{122}^1 + 7x_{132}^1 \\ +8x_{212}^1 + 6x_{222}^1 + 16x_{232}^1 + 7x_{111}^2 + 7x_{121}^2 + 12x_{131}^2 + 10x_{211}^2 + 7x_{221}^2 \\ +13x_{231}^2 + 11x_{112}^2 + 8x_{122}^2 + 8x_{132}^2 + 11x_{212}^2 + 8x_{222}^2 + 16x_{232}^2 \end{array} \right) \]

\[ \left( c_{ijk}^{2p} \right)^U = \left( \begin{array}{c} 7x_{111}^1 + 6x_{121}^1 + 10x_{131}^1 + 9x_{211}^1 + 7x_{221}^1 + 9x_{231}^1 + 8x_{112}^1 + 7x_{122}^1 + 9x_{132}^1 \\ +8x_{212}^1 + 11x_{222}^1 + 11x_{232}^1 + 9x_{111}^2 + 9x_{121}^2 + 14x_{131}^2 + 13x_{211}^2 + 8x_{221}^2 \\ +15x_{231}^2 + 13x_{112}^2 + 10x_{122}^2 + 12x_{132}^2 + 14x_{212}^2 + 10x_{222}^2 + 18x_{232}^2 \end{array} \right) \]

\[ \left( d_{ijk}^{2p} \right)^L = \left( \begin{array}{c} 7x_{111}^1 + 5x_{121}^1 + 12x_{131}^1 + 10x_{211}^1 + 7x_{221}^1 + 13x_{231}^1 + 11x_{112}^1 + 8x_{122}^1 + 6x_{132}^1 \\ +11x_{212}^1 + 8x_{222}^1 + 16x_{232}^1 + 1x_{111}^2 + 2x_{121}^2 + 7x_{131}^2 + 10x_{211}^2 + 13x_{221}^2 \\ +7x_{231}^2 + 9x_{112}^2 + 8x_{122}^2 + 8x_{132}^2 + 9x_{212}^2 + 7x_{222}^2 + 13x_{232}^2 \end{array} \right) \]

The steps of the solution procedure as follow:

1. \( \text{(Min}(Z_1)^U)_{\text{max}} \) (Min}(Z_1)^U)_{\text{min}} \left( \text{Min}(Z_2)^U \right)_{\text{max}} = 4.5 (\text{Min}(Z_2)^U)_{\text{min}}

2. The membership functions are:

\[ \mu_1((Z_1(x))^U) = \begin{cases} 1, & (Z_1(x))^U \leq 0.9375, \\ \frac{4.5 - (Z_1(x))^U}{3.5625}, & 0.9375 \leq (Z_1(x))^U < 4.5, \\ 0, & (Z_1(x))^U \geq 4.5, \end{cases} \]

\[ \mu_2((Z_2(x))^U) = \begin{cases} 1, & (Z_2(x))^U \leq 0.615, \\ \frac{4.5 - (Z_2(x))^U}{3.885}, & 0.615 \leq (Z_2(x))^U < 4.5, \\ 0, & (Z_2(x))^U \geq 4.5, \end{cases} \]

3. Solve the problem corresponding to problem (16) as

\[
\text{Max } \lambda \\
\text{Subject to }
\]
The solution is
\[
x_{121}^1 = 1.38, x_{132}^1 = 0.14, x_{111}^2 = 0.90,
\]
\[
Z_1 = [1.3225, 2.734], Z_2 = [0.74, 1.785].
\]
7. Concluding remarks

In this paper, fractional multi-objective multi-product solid transportation problem with interval costs, supply, demand, and conveyances has been investigated based on fuzzy programming approach. The advantages are that the problem with interval-valued allows the DM to deal with a situation realistically. To deal with the minimization problem, the order relations who represent the decision maker's (DM) performance between interval costs, supply, demand and conveyances has been defined by the right limit, the left limit, the center and the width of intervals. Through the deterministic problem is obtained, a fuzzy compromise approach has been applied by defining membership functions. A linear membership function has been used for obtaining optimal compromise solution.

References


