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An Economical Acceptance-Sampling Plan Based on Binomial Distribution with Imperfect Inspection

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Abstract. One of the most useful and effective methods with an extensive application in companies with the purpose of examining the quality of the raw material in addition to final products, is acceptance sampling plans. The inspection process is assumed to be free of errors in most of the acceptance sampling plans. However, this assumption may not be true. In this research, an optimization model for acceptance sampling plan is developed based on Bayesian inferences in the presence of inspection errors. An economical model is designed which involves two types of inspection errors and investigates the impact of these errors on the acceptance-sampling plan. Furthermore, the sensitivity analysis is carried out to analyze the behavior of optimal solution. The numerical studies indicate that increasing the inspection error leads to decrease the sample size and acceptance number.

Keywords: Acceptance Sampling Plan, Bayesian Inferences, Economic, Inspection, Inspection Errors.

1. Introduction

Acceptance sampling methods have been applied in different directions for the inspection and testing the raw material or the final products. Since it may be impossible to inspect or test each item in the production environments, thus acceptance sampling plan provides confidence to the producer and consumer that the products are conforming to the given specifications, and it decreases time and cost of the inspection. In this article, an acceptance-sampling plan is developed to decide about the lot based on the cost objective function in the presence of inspection errors. Two concepts of type I and type II Inspection errors, and two decisions including accepting the lot or rejecting the lot are considered in the cost objective function. Inspection errors affect on the performance measures of the sampling plan. The source of these errors can be operational environment, inspector fatigue, and failure of inspection tool. Therefore, it is necessary to analyze the statistical and economic influence of inspection errors on the performance measures of a sampling plan. Khan and Duffuaa (2002) analyzed the effect of inspection errors on the result of different inspection plans. Duffuaa (1996) studied the statistical and economic effect of inspector errors on sampling plan. Tang and Schneider (1987) analyzed the economic and statistical effect of inspection error on the rectifying sampling plan. They developed two models in the presence of inspection errors under different rework plans. Raoufet *et al.* (1983) developed a model to determine the optimal sample size for multi-characteristic elements to minimize the total expected cost per unit item by considering Type I error, Type II error and the cost of inspection. Bennetet *et al.* (1974) studied a single sampling plan in the presence of inspection error with known incoming process mean. Collins *et al.* (1973) investigated the behavior of inspection error on the probability of acceptance, average total inspection and average outgoing quality. They studied the sampling plan under both replacement and non-replacement rectifying policies. Markowski and Markowski (2002) studied an attribute acceptance-sampling plan in the presence of inspection errors and introduced new sampling plans to consider the risk of statistical classification error. Their analysis denoted that there are important shortcomings in traditional sampling plans. Ferrell and Chhoker (2002) developed models for 100% inspection and single sampling, with and without inspection error using Taguchi-like loss function. Arshadi Khamseh *et al.* (2008) developed an economical model for double variable acceptance sampling plan in the presence of inspection errors. They considered Taguchi Loss function as the acceptance cost while quality characteristics follow normal distribution with known variance. Fallah Nezhad and Hosseini Nasab (2011) used a control threshold policy to design an optimum acceptance sampling plan. Fallah Nezhad and Hosseini Nasab (2012) presented a new acceptance-sampling plan when the inspection process is imperfect. They developed a Bayesian technique for determining the probability density function of the number of defective items. They showed that negative binomial prior is a suitable distribution for modeling the Bayesian acceptance-sampling plan. Fallahnezhad and Aslam (2013) represented a new economical design of acceptance sampling plan using Bayesian inference in order to decide about the received lot based on cost objective function. They used Bayesian inference to determine the optimal decision based on backward induction. Fallahnezhad and Niaki (2013) and Fallahnezhad *et al.* (2012) investigated economic models for sampling plan. Aslam *et al.* (2013) and Fallahnezhad and Niaki (2011) proposed different methodologies for economical design of sampling plan in production environment. The remaining of this paper is organized as follows: In section 2, we present the abbreviations and notations. Then we provide performance measures. The cost minimization model is introduced in section 3. Section 4 provides numerical example and discussion. Sensitivity analyses are given in section 5. Finally, in section 6, the conclusions are presented.

2. The Preliminaries

Mathematical formulation of the acceptance sampling plan can help for investigating the influence of the inspection errors on the acceptance sampling plan. The following notations are used throughout the paper.

AQL: Acceptance Quality Level
LQL: Limiting Quality Level
N: the total number of items in a lot

n: sample size
c: acceptance number
p: the nonconforming proportion
k: the number of nonconforming items in the sample of size n
j: the number of nonconforming items in the N-n remaining items
I_1: *the cost of inspecting one item*
I_2: *the cost of replacement or repair for one nonconforming item during the inspection*
I_3: *the cost of classifying one conforming item as nonconforming*
A_2: *the cost of classifying one nonconforming item as conforming item (Warranty cost)*
α: *Producre's risk*
β: *Consumer's risk*

The objective function is the sum of the expected cost of accepting the lot plus the expected cost of rejecting the lot. In an acceptance-sampling plan, a sample size of n is selected from a lot with size N and each item in the sample is inspected and classified as either conforming or nonconforming. If the number of nonconforming items is more than the acceptance number c, then the whole lot is inspected using rectifying inspection policy. Otherwise, it is accepted. The inspection errors in an attribute sampling are categorized into two types. Type I error is to classify a conforming item as non-conforming, and Type II error is to classify a nonconforming item as conforming. Thus, the following results are obtained:

$e_1 = P$ *{the item is classified as nonconforming| the item is conforming}*
$e_2 = P$ *{the item is classified as conforming| the item is nonconforming}*
$e_1 =$ *Type I error probability*
$e_2 =$ *Type II error probability*

Then, the apparent nonconforming proportion p' is obtained as follows:

$A =$ *the event that an item is nonconforming*
$B =$ *the event that an item is classified as nonconforming*

$$p' = p(B) = p(B|A)p(A) + p(B|A')p(A') = (1 - e_2)p + e_1(1 - p) \qquad (1)$$

Where

p= p (A), true nonconforming fraction
P′= p (B), apparent nonconforming fraction

In addition, the apparent AQL and LQL are obtained as follows:

$$\begin{aligned} AQL' &= (1-e_2)AQL \ + \ e_1(1-AQL) \\ LQL' &= (1-e_2)LQL \ + \ e_1(1-LQL) \end{aligned} \tag{2}$$

In addition, following conditional probabilities are obtained using Bayesian rule:

$$s = p\{\textit{the item is nonconforming} \mid \textit{the item is classified as nonconforming}\}$$

$$s = \frac{p(B|A)p(A)}{p(B|A)p(A) + p(B|A')p(A')} = \frac{(1-e_2)p}{(1-e_2)p + e_1(1-p)} \tag{3}$$

$$d = p\{\textit{the item is nonconforming} \mid \textit{the item is classified as conforming}\}$$

$$d = \frac{p(B'|A)p(A)}{(p(B'|A)p(A) + p(B'|A')p(A')} = \frac{e_2.p}{(e_2.p + (1-e_1)(1-p)\,)} \tag{4}$$

3. Cost Minimization Model

We have tried to design the optimal sampling plan by minimizing the summation of the cost of inspection and cost of nonconforming items and total cost of misclassification resulted from type I and II errors. The mathematical formulation for expected total cost will be formulated by considering different events. The probabilities of type I and II errors are assumed to be known. The optimization model in the presence of inspection errors is modeled as follows:

$$Minimize\ Z = \sum_{k=0}^{n}\{\sum_{j=0}^{N-n} (I_1.n+I_2.k.s+I_3.k(1\text{-}s)+A_2(j+(n\text{-}k).d))\binom{N-n}{j}p^j(1\text{-}p)^{N-n-j}\}\binom{n}{k}p'^k(1\text{-}p')^{n-k}+\sum_{k=c+1}^{n}\{\sum_{j=0}^{N-n} (I_1.N+I_2(k.s+j(1\text{-}e_2))+I_3(k(1\text{-}s)+(N\text{-}n\text{-}j)e_1)+A_2(je_2+(n\text{-}k).d))\binom{N-n}{j}p^j(1\text{-}p)^{N-n-j}\}\binom{n}{k}p'^k(1\text{-}p')^{n-k} \tag{5}$$

S.t:

$$\sum_{k=0}^{n} \binom{n}{k} AQL'^k(1\text{-}AQL')^{n-k} \geq 1\text{-}\alpha$$

$$\sum_{k=0}^{n} \binom{n}{k} LQL'^k(1\text{-}LQL')^{n-k} \leq \beta$$

It is assumed to apply rectifying inspection when the lot is rejected. Thus, when we reject the lot then all items are inspected. The cost terms for the decision of accepting the lot are elaborated as following:

$I_1.n$: this term denotes the cost of inspecting n items

$I_2.k.s$: this term denotes the cost of replacement or repair k×s items that are classified as nonconforming and they are really nonconforming

$I_3.k.(1\text{-}s)$: this term denotes the cost of k (1-s) items that are classified as nonconforming but they are conforming

$A_2.(j+(n\text{-}k).d)$: this term denotes the cost of classifying j+(n-k)d nonconforming items as conforming in the accepted lot

The components of rejection cost are expressed as following:

$I_1.N$: this term denotes the cost of inspecting N items

$I_2(k.s+j(1\text{-}e_2))$: this term denotes the cost of replacement or repair for $ks+j(1\text{-}e_2)$ nonconforming items classified as nonconforming

$I_3(k(1\text{-}s)+(N\text{-}n\text{-}j)e_1)$: this term denotes the cost of $k(1\text{-}s)+(N\text{-}n\text{-}j)e_1$ conforming items that are classified as nonconforming

$A_2(je_2+(n\text{-}k).d)$: this term denotes the cost of classifying $je_2+(n\text{-}k)d$ nonconforming items that are classified as conforming

4. Numerical Example and Discussion

The following numerical example is studied to illustrate the application of the proposed methodology.

For more illustrations, assume that the following set of input parameters is given:

$$I_1=1000,\ I_2=1500,\ I_3=3000,\ A_2=5000,\ N=90,$$
$$p=0.05,\ AQL=0.01,\ LQL=0.2,\ \alpha=0.1,\ \beta=0.2.$$

Also a grid search procedure is employed for determining the optimal solution in the intervals n={1,2,…,90} and c={0,1,…,n}. The question is to find the minimum total cost by determining the optimal values of n and c such that the constraints of producer's risk and the consumer' risk are satisfied simultaneously for the given values of AQL and LQL. The MATLAB software has been used in order to obtain the optimal solution. First we assume specified intervals for n, c in order to search for determining the optimal solution. Then a grid search procedure is applied to obtain the optimal solution. The main goal is to obtain the minimum total cost by determining the optimal values of n and c within those assumed interval for each decision variable. The procedure of obtaining the minimum total cost is as follows:

The optimal values for n and c should been searched within the given assumed interval. Each value of decision variable in the set which could satisfy the constraints of producer's and consumer's risks constraints simultaneously, is chosen and then the feasible values of n and c are substituted in the objective function in order to achieve the minimum total cost.

The obtained result is as follows: n=19; c=2 and cost=47902

5. Sensitivity Analysis

In this section, a sensitivity analysis is conducted to investigate the behavior of optimal solution concerning the variation of parameters.

Table 1. The sensitivity analysis for different value I_1

I_1	n	c	Cost
1000	19	2	47902
200	19	2	24904
100	90	5	21019

Table 2. The sensitivity analysis for different value I_2

I_2	n	c	Cost
1000	19	2	47190
1500	19	2	47902
5000	19	2	52884

Table 3. The sensitivity analysis for different value I_3

I_3	n	c	Cost
1500	19	2	47083
3000	19	2	47902

5000	19	2	48993

Table 4. The sensitivity analysis for different value A_2

A_2	n	c	Cost
1500	19	2	37134
5000	19	2	47902
10000	19	2	63283
50000	90	15	103900

Table 5. The sensitivity analysis for different value α

α	n	c	Cost
0.01	25	3	49645
0.1	19	2	47902
0.2	19	2	47902

Table 6. The sensitivity analysis for different value β

β	n	c	Cost
0.1	30	3	56776
0.2	19	2	47902
0.3	11	1	44174

It is observed that when the value of risks α and β decreases then the optimal value of sample size, acceptance number and total cost of sampling plan increase.The parameters I_2 and I_3 have little effect on the optimal solution but the variations of I_1 and A_2 can completely change the optimal solution.

6. Conclusion

The acceptance sampling plan is one of the significant methods which is applied to evaluate the quality of the raw material, semi-finished products and final goods and it is used in almost any kind of industry. In this article, an acceptance-sampling plan is developed to decide about the lot based on the cost objective function in the presence of inspection errors. Two concepts of type I and type II Inspection errors, and two decisions including accepting the lot or rejecting the lot are considered in the cost objective function. Inspection errors affect on the performance measures of the sampling plan. The source of these errors can be operational environment, inspector fatigue, and failure of inspection tool. Therefore, it is necessary to analyze the statistical and economic influence of inspection errors on the performance measures of a sampling plan. The result of sensitivity analysis denotes that when the value of risks decreases then optimal value of sample size and acceptance number increase.

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