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An Economic Design of Combined Double Sampling and Variable Sample Size $\bar{\mathbf{X}}$ Control Chart

Saeed Khaki
Department of Industrial Engineering and Management Systems,
Amirkabir University of Technogy, Tehran, Iran
(Corresponding Author)

Niloufar Ghanbari
Department of Industrial Engineering and Management Systems,
Amirkabir University of Technogy, Tehran, Iran
n_ghanbari@aut.ac.ir

Mir Mahdi Seyed Esfehani
Department of Industrial Engineering and Management Systems,
Amirkabir University of Technogy, Tehran, Iran
msesfahani@aut.ac.ir

Abstract. In recent years several studies have shown that $\bar{X}$ control charts with adaptive schemes or double sampling plans detect both small and moderate shifts in the process mean more quickly than the traditional Shewhart $\bar{X}$ chart. In the classical double sampling $\bar{X}$ chart, the difference between two points were placed in the central region of first stage was not considered. In this study, a new control chart is proposed by combination of double sampling $\bar{X}$ chart and variable sample size $\bar{X}$ chart (called DSVSS chart), that can successfully reduce the detection time of small mean shift. Before a DSVSS $\bar{X}$ chart is used, its design parameters should be determined, hence economic design model of DSVSS $\bar{X}$ chart is constructed. Markov chain approach is used to compute the statistical properties of the chart that are essential to our cost function. Then, the Genetic Algorithms (GA) are used to solve the optimal designs of DSVSS $\bar{X}$ chart. Finally, a numerical example is provided to illustrate the use of this model.

Keywords; Quality Control, Double Sampling Chart, Variable Sample Size Chart, Markov Chain, Genetic Algorithms.

1. Introduction

Control chart is a main statistical process control (SPC) tool to detect the occurrence of assignable causes so that a remedial action can be taken before many defective products are manufactured in a process. Dr.shewhart created $\bar{X}$ control chart, which $\mu \pm 3\sigma_{\bar{X}}$ is used to set control limits for controlling process. It is easy for operators to understand how Shewhart's control chart works, so the chart is widely applied in industries. Static strategies have become less and less suitable for today's advanced industrial society, because of their low performance in detecting small process shifts quickly. Researchers have been trying to propose various adaptive schemes in control chart in order to respond to a shift in process immediately, when a control chart is used to monitor a process, three design parameters the operator must select are the sample size, the sampling interval, and the action limit. All the three parameter of Shewhart control charts are constant. Studies have shown that we can improve performance of control charts by changing their parameters within the production controlling. We can categorize the adaptive schemes the following types: variable sample size (VSS) chart, variable sample interval (VSI) chart, variable control limit (VCL) chart,and joint-adaptive charts which at least two of the designing parameters are variables. Adaptive control charts was first proposed by Reynolds et al. (1988). They used the $\bar{X}$ control chart with variable sampling intervals (VSI) and used average time to signal and average number of sample to signal as performance measure and showed that their control chart works better than Shewhart control chart. They concluded a shorter sample interval is used if there is sign that the process might have changed and the longer sample interval is used if there is no such indication. In the industries, the nature of the process may need that the sampling frequency must be fixed, but there is no limitation for the sample size, therefore the VSS scheme might be more reasonable than VSI scheme from an industrial perspective. Prabhu *et al.* (1993) and Costa (1994) proposed the VSS schemes for $\bar{X}$ control chart. They investigated a smaller sample size for next sample when current sample $\bar{X}$ value is close to center line and larger sample size in otherwise. Costa (1997, 1998, 1999) presented a fully adaptive and R chart. Costa also showed that the VP $\bar{X}$ and R chart has better efficiency than other adaptive $\bar{X}$ and R charts and the joint Shewhart $\bar{X}$ and R chart. The VP $\bar{X}$ and R charts plot the sample means on the $\bar{X}$ chart and sample ranges are plotted on the R chart. Daudin (1992) proposed a double sampling (DS) control chart determine by two sample sizes where a second sample is inspected only if the first is not sufficient to make a decision about the state of the process. Carot et al. (2002) further combined the double sampling method with the VSI $\bar{X}$ chart. Khoo et al. (2010) combined the synthetic chart and DS $\bar{X}$ chart to increase the detecting efficiency in process mean shifts. Lee et al. (2012b) designed the DSVSI $\bar{X}$ chart with an economic design method. Many researchers have considered an economic criteria in the design of a control chart. The economic design includes developing a cost function that considers all the cost components relevant to monitoring and controlling a process. The design parameters are selected such that the cost function is minimized. This method has been widely called the economic design of control charts in the quality control literature (Montgomery, 1980; Prabhu et al., 1997). Duncan (1956) constructed a cost model for the design parameters (sample size, sampling interval, and control limit coefficient) of Shewhart's $\bar{X}$ control chart. Lorenzen and Vance (1986) developed one unified approach to the economic design of control charts. They constructed a general cost model that applied to all control charts. Elsayed (1994) developed an economic design of $\bar{X}$ control chart using quadratic loss function. In this study, a new chart is proposed to improve the performance of double sampling (DS) control chart by combination of double sampling $\bar{X}$ chart and variable sample size $\bar{X}$ chart (DSVSS). In classical double sampling $\bar{X}$chart, there was not any difference between two points were placed in the in-control region (central). Hence, VSS scheme is combined into first stage of double sampling $\bar{X}$ chart in order to improve this weakness.

Before a DSVSS $\bar{X}$ chart is used, its design parameters should be determined.we develop an economic design of the $\bar{X}$ control chart based on Markov chain approach. The use of Markov chain allows us easily to obtain the statistical properties of the chart that are necessary to our cost function. The optimal solution can be found through the genetic algorithms (GAs) such that the cost function is minimized. Finally, numerical illustration of DSVSS $\bar{X}$ chart is performed.

2. Combined Double Sampling and Variable Sample Size $\bar{\mathbf{X}}$ Control Chart

A combination of double sampling $\bar{X}$ chart and variable sample size $\bar{X}$ chart (DSVSS) includes 8 design parameters such as n_1, n_2, n_3, h, w_1, k_1 and k_2. The parameters n_1 and n_2 are respectively the sample sizes ($n_2 > n_1$) of the first stages and n_3 is the sample size of second stage of the double sampling. h is the sampling interval, w_1 and w_2 are the warning limits to change the sample size, and k_1 and k_2 are the control limit coefficients of the first and second stages of the double sampling. For the normally distributed observations, we can construct the DSVSS as Figure. 1. The operational procedure for the DSVSS is, first take a small sample of size n_1, and calculate its sample mean $\bar{X}_1$. If $\bar{X}$ falls in I_1, the process will be considered as an in-control state, and the next sample size is n_1, if $\bar{X}$ falls in I_2, the next sample size is n_2, if $\bar{X}$ falls in I_4, the process will be considered as an out-of-control state. For the case that $\bar{X}$ falls in I_3, it is necessary to take a second sample of size n_3 with a sample mean $\bar{X}_3$ and continue monitoring the process with the second-stage chart. The total sample mean $\bar{Y}$ can then be computed, if previous point was placed in I_1 with

$$\bar{Y} = \frac{n_1\bar{X}_1 + n_3\bar{X}_3}{n_1 + n_3},$$

or if previous point was placed in I_2 with

$$\bar{Y} = \frac{n_2\bar{X}_2 + n_3\bar{X}_3}{n_2 + n_3}.$$

If $\bar{Y}$ falls in I_5, the process will be considered in control and we will start the monitoring with sample size n_1 again .Otherwise, the process will be regarded in an out-of-control state. After a false alarm, the first sample size n_1 is taken.

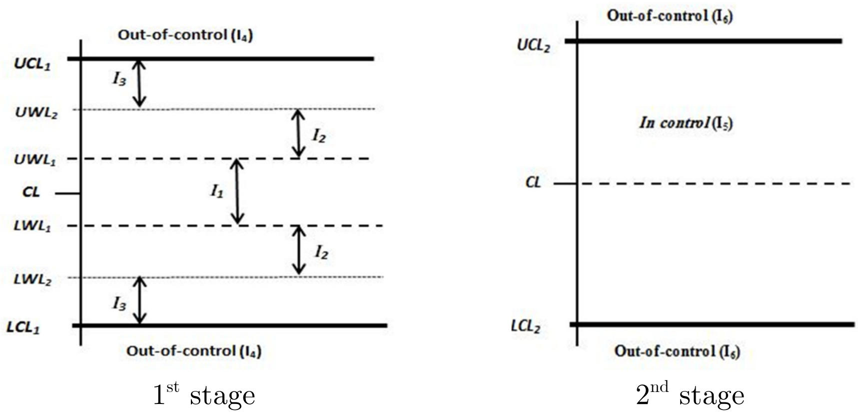

Figure 1. DSVSS control chart.

$$UCL_1 = \mu_0 + k_1 \qquad LCL_1 = \mu_0 - k_1 \qquad UWL_1 = \mu_0 + w_1$$
$$UWL_2 = \mu_0 + w_2 \qquad LWL_1 = \mu_0 - w_1 \qquad LWL_2 = \mu_0 - w_2$$
$$UCL_2 = \mu_0 + k_2 \qquad LCL_2 = \mu_0 - k_2 \qquad Cl = \mu_0,\ i = 1,2$$

3. The Construction of an Economic Design Model

In this section, we construct a cost function for the design optimization of DSVSS. In our economic model, a process is assumed to start with an in-control state ($\mu = \mu_0$) but after a random time, single assignable cause will be occurred, that causes a fixed shift in the process mean ($\mu = \mu_1$). After the shift, the process remains out-of-control until the assignable cause is deleted (if possible). The time interval that process remains in control is exponential random variable with parameter λ and the time interval between two sampling stages is 0. A sample is taken at each sampling time to compute the $\bar{X}$ value. If $\bar{X}$ falls inside the action limit, its value will be used to determine the next sample size. Otherwise, the process is stopped and a search starts to find cause and repair the process. Production cycle considered in the cost model is shown in Figure. 2.

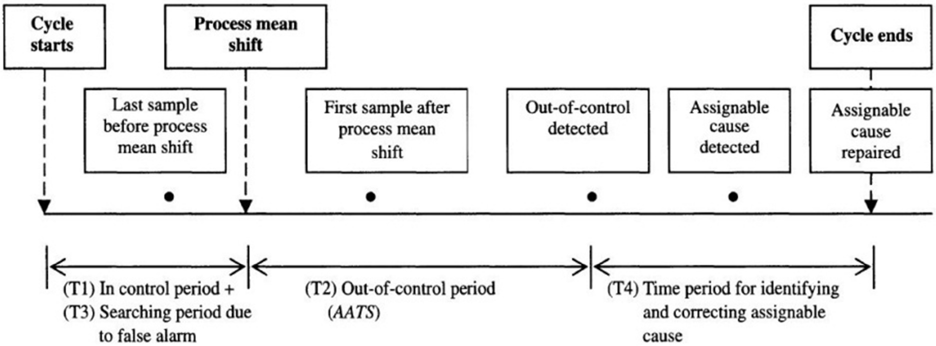

Figure 2. The production cycle

Figure 2 shows the production cycle, which is divided into four time intervals of in-control period (T_1), out-of-control period (T_2), searching period due to false alarm (T_3), and the time period for identifying and repairing the assignable cause (T_4).

T_1: The expected length of in-control period is $1/\lambda$.

T_2: The expected length of out-of-control shows the average time needed for the control chart to produce a signal after the process mean shift. This average time is the adjusted average time to signal (AATS).The memory less property of the exponential distribution permits the computation of AATS using the Markov Chain approach.

Let M be the average time from the cycle start to the time the chart signals after the process shift. Then,

$$AATS = M - \frac{1}{\lambda}$$

According to the status of the process (in or out-of-control) and the position of $\bar{X}_i$, we have eight transient states. Shift size is denoted by δ. $(\mu_1 = \mu_0 + \delta)$

For ith sampling, the states of the Markov Chain are:

State 1: the process is in-control (δ_i=0) and $\bar{X}_i$ falls into the I_1

State 2: the process is in-control (δ_i=0) and $\bar{X}_i$ falls into the I_2

State 3: the process is in-control (δ_i=0) and $\bar{X}_i$ falls into the I_3

State 4: the process is in-control (δ_i=0) and $\bar{X}_i$ falls into the I_4

State 5: the process is out-of-control ($\delta_i \neq 0$) and $\bar{X}_i$ falls into the I_1

State 6: the process is out-of-control ($\delta_i \neq 0$) and $\bar{X}_i$ falls into the I_2

State 7: the process is out-of-control ($\delta_i \neq 0$) and $\bar{X}_i$ falls into the I_3

State 8: the process is out-of-control ($\delta_i \neq 0$) and $\bar{X}_i$ falls into the I_4

When State 4 is reached, the signal the chart gives is a false alarm. If $\bar{X}$ falls into the action region at a sampling time while the process status is out-of-control, then the signal is a true alarm and the absorbing state, State 8, is reached.

The transition probability matrix is:

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} & P_{18} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & P_{37} & P_{38} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & P_{46} & P_{47} & P_{48} \\ 0 & 0 & 0 & 0 & P_{55} & P_{56} & P_{57} & P_{58} \\ 0 & 0 & 0 & 0 & P_{65} & P_{66} & P_{67} & P_{68} \\ 0 & 0 & 0 & 0 & P_{75} & P_{76} & P_{77} & P_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where p_{ij} represents the transition probability that i is the prior state and j is the current state. Then we have:

$$P_{11} = Pr(\bar{X}_i \in I_1,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0) = e^{-\lambda h}[2\phi(w_1) - 1]$$

$$P_{12} = Pr(\bar{X}_i \in I_2,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0) = e^{-\lambda h}[2[\phi(w_2) - \phi(w_1)]]$$

$$P_{13} = Pr(\bar{X}_i \in I_3,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0) = e^{-\lambda h}[2[\phi(k_1) - \phi(w_2)]$$

$$P_{14} = Pr(\bar{X}_i \in I_4,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0) = e^{-\lambda h}[2 - 2\phi(k_1)]$$

$$P_{15} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0)$$

$$= (1 - e^{-\lambda h})[\phi(-\delta\sqrt{n_1} + w_1) - \phi(-\delta\sqrt{n_1} - w_1)]$$

$$P_{16} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[\phi(w_2-\delta\sqrt{n_1})-\phi(w_1-\delta\sqrt{n_1})+\phi(-\delta\sqrt{n_1}-w_1)-\phi(-\delta\sqrt{n_1}-w_2)]$$
$$P_{17} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[\phi(-\delta\sqrt{n_1}+k_1)-\phi(-\delta\sqrt{n_1}+w_2)+\phi(-\delta\sqrt{n_1}-w_2)-\phi(-\delta\sqrt{n_1}-k_1)]$$
$$P_{18} = Pr(\bar{X}_i \in I_4,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[1-\phi(k_1-\delta\sqrt{n_1})+\phi(-k_1-\delta\sqrt{n_1})]$$
$$P_{21} = Pr(\bar{X}_i \in I_1,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0) = e^{-\lambda h}[2\phi(w_1)-1]$$
$$P_{22} = Pr(\bar{X}_i \in I_2,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0) = e^{-\lambda h}[2[\phi(w_2)-\phi(w_1)]]$$
$$P_{23} = Pr(\bar{X}_i \in I_3,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0) = e^{-\lambda h}[2(\phi(k_1)-\phi(w_2)]$$
$$P_{24} = Pr(\bar{X}_i \in I_4,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0) = e^{-\lambda h}[2-2\phi(k_1)]$$
$$P_{25} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[\phi(w_1-\delta\sqrt{n_2})-\phi(-\delta\sqrt{n_2}-w_1)]$$
$$P_{26} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[\phi(w_2-\delta\sqrt{n_2})-\phi(w_1-\delta\sqrt{n_2})+\phi(-\delta\sqrt{n_2}-w_1)-\phi(-\delta\sqrt{n_2}-w_2)]$$
$$P_{27} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[\phi(-\delta\sqrt{n_2}+k_1)-\phi(-\delta\sqrt{n_2}+w_2)+\phi(-\delta\sqrt{n_2}-w_2)-\phi(-\delta\sqrt{n_2}-k_1)]$$
$$P_{28} = Pr(\bar{X}_i \in I_4,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} = 0)$$
$$= (1-e^{-\lambda h})[1-\phi(k_1-\delta\sqrt{n_2})+\phi(-k_1-\delta\sqrt{n_2})]$$

Let $\bar{n}$ be the average sample size is taken from first stage to go second stage, thus:

$$\bar{n} = n_1\left(p_{13} + p_{53}\right) + n_2\left(p_{23} + p_{63}\right)$$

We assume if $\bar{X}$ falls in I_3, then $\bar{Y}$ falls in I_5, we start the monitoring the process with sample size n_1 in first stage.

$$P_{31} = Pr(\bar{X}_i \in I_1,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0) \times pr(\bar{Y} \in I_5)$$
$$= e^{-\lambda h}[2\phi(w_1)-1] \times [2\phi(k_2)-1]$$
$$P_{32} = Pr(\bar{X}_i \in I_2,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0) \times pr(\bar{Y} \in I_5)$$

$$= e^{-\lambda h}[2[\phi(w_2)-\phi(w_1)]]\times[2\phi(k_2)-1]$$

$$P_{33} = Pr(\bar{X}_i \in I_3,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0)\times pr(\bar{Y} \in I_5)$$

$$= e^{-\lambda h}[2[\phi(k_1)-\phi(w_2)]]\times[2\phi(k_2)-1]$$

$$P_{34} = Pr(\bar{Y}_i \in I_6,\ \delta_i = 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0)$$

$$= e^{-\lambda h} Pr(\bar{Y}_i \in I_6,\ \delta_i = 0) = e^{-\lambda h}[2-2\phi(k_2)]$$

$$P_{35} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0)\times pr(\bar{Y} \in I_5)$$

$$= (1-e^{-\lambda h})[\phi(w_1-\delta\sqrt{n_1})-\phi(-w_1-\delta\sqrt{n_1})]\times[2\phi(k_2)-1]$$

$$P_{36} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0)\times pr(\bar{Y} \in I_5)$$

$$= (1-e^{-\lambda h})[\phi(w_2-\delta\sqrt{n_1})-\phi(w_1-\delta\sqrt{n_1})+\phi(-\delta\sqrt{n_1}-w_1)-\phi(-\delta\sqrt{n_1}-w_2)]$$
$$\times[2\phi(k_2)-1]$$

$$P_{37} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} = 0)\times pr(\bar{Y} \in I_5)$$

$$= (1-e^{-\lambda h})[\phi(-\delta\sqrt{n_1}+k_1)-\phi(-\delta\sqrt{n_1}+w_2)+\phi(-\delta\sqrt{n_1}-w_2)-\phi(-\delta\sqrt{n_1}-k_1)]$$
$$\times[2\phi(k_2)-1]$$

$$P_{38} = Pr(\bar{Y}_i \in I_6,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_i = 0)$$

$$= (1-e^{-\lambda h})[1-\phi(k_2-\delta\sqrt{\bar{n}+n_3})+\phi(-k_2-\delta\sqrt{\bar{n}+n_3})]$$

$$P_{41} = P_{11};\ P_{42} = P_{12};\ P_{43} = P_{13};\ P_{44} = P_{14};\ P_{45} = P_{15};\ P_{46}$$
$$= P_{16};\ P_{47} = P_{17};\ P_{48} = P_{18}$$

$$P_{51} = P_{52} = P_{53} = P_{54} = 0$$

$$P_{55} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} \neq 0) = \phi(w_1-\delta\sqrt{n_1})-\phi(-\delta\sqrt{n_1}-w_1)$$

$$P_{56} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} \neq 0)$$

$$= \phi(w_2-\delta\sqrt{n_1})-\phi(w_1-\delta\sqrt{n_1})+\phi(-\delta\sqrt{n_1}-w_1)-\phi(-\delta\sqrt{n_1}-w_2)$$

$$P_{57} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} \neq 0)$$

$$= [\phi(k_1-\delta\sqrt{n_1})-\phi(w_2-\delta\sqrt{n_1})+\phi(-\delta\sqrt{n_1}-w_2)-\phi(-\delta\sqrt{n_1}-k_1)]$$

$$P_{58} = Pr(\bar{X}_i \in I_4,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_1,\ \delta_{i-1} \neq 0)$$

$$= [1-\phi(k_1-\delta\sqrt{n_1})+\phi(-k_1-\delta\sqrt{n_1})]$$

$$P_{61} = P_{62} = P_{63} = P_{64} = 0$$

$$P_{65} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_i \neq 0) = \phi(w_1 - \delta\sqrt{n_2}) - \phi(w_1 - \delta\sqrt{n_2})$$

$$P_{66} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_i \neq 0)$$
$$= \phi(w_2 - \delta\sqrt{n_2}) - \phi(w_1 - \delta\sqrt{n_2}) + \phi(-\delta\sqrt{n_2} - w_1) - \phi(-\delta\sqrt{n_2} - w_2)$$

$$P_{67} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} \neq 0)$$
$$= [\phi(k_1 - \delta\sqrt{n_2}) - \phi(w_2 - \delta\sqrt{n_2}) + \phi(-\delta\sqrt{n_2} - w_2) - \phi(-\delta\sqrt{n_2} - k_1)$$

$$P_{68} = Pr(\bar{X}_i \in I_4,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_2,\ \delta_{i-1} \neq 0)$$
$$= 1 - \phi(k_1 - \delta\sqrt{n_2}) + \phi(-k_1 - \delta\sqrt{n_2})$$

$$P_{71} = P_{72} = P_{73} = P_{74} = 0$$

$$P_{75} = Pr(\bar{X}_i \in I_1,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} \neq 0) \times pr(\bar{Y} \in I_5)$$
$$= [\phi(w_1 - \delta\sqrt{n_1}) - \phi(-w_1 - \delta\sqrt{n_1})] \times [\phi(k_2 - \delta\sqrt{\bar{n} + n_3}) - \phi(-\delta\sqrt{\bar{n} + n_3} - k_2)]$$

$$P_{76} = Pr(\bar{X}_i \in I_2,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} \neq 0) \times\ Pr(\bar{Y} \in I_5)$$
$$= [\phi(w_2 - \delta\sqrt{n_1}) - \phi(w_1 - \delta\sqrt{n_1}) + \phi(-\delta\sqrt{n_1} - w_1) - \phi(-\delta\sqrt{n_1} - w_2)]$$
$$\times [\phi(k_2 - \delta\sqrt{\bar{n} + n_3}) - \phi(-\delta\sqrt{\bar{n} + n_3} - k_2)]$$

$$P_{77} = Pr(\bar{X}_i \in I_3,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_{i-1} \neq 0) \times pr(\bar{Y} \in I_5)$$
$$= [\phi(-\delta\sqrt{n_1} + k_1) - \phi(-\delta\sqrt{n_1} + w_2) + \phi(-\delta\sqrt{n_1} - w_2) - \phi(-\delta\sqrt{n_1} - k_1)]$$
$$\times [\phi(k_2 - \delta\sqrt{\bar{n} + n_3}) - \phi(-\delta\sqrt{\bar{n} + n_3} - k_2)]$$

$$P_{78} = Pr(\bar{Y}_i \in I_6,\ \delta_i \neq 0 \,|\, \bar{X}_{i-1} \in I_3,\ \delta_i \neq 0)$$
$$= [1 - \phi(k_2 - \delta\sqrt{\bar{n} + n_3}) - \phi(-k_2 - \delta\sqrt{\bar{n} + n_3})]$$

$$P_{81} = P_{82} = P_{83} = P_{84} = P_{85} = P_{86} = P_{87} = 0;\ P_{88} = 1$$

Where $\phi(\ )$ is the cumulative distribution function of a standard normal distribution.

According to the primary properties of Markov Chains (Cinlar, 1975), we know that $r'(I - Q)^{-1}$, shows the mean number of transitions in each transient state before the true alarm signals, where $r' = (r_1, r_2, r_3, r_4, r_5, r_6, r_7)$, is the vector of starting probability that $r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7 = 1$; I is identity matrix of order 7; Q is the transition matrix where the elements associated with the absorbing state have been removed.

Let M be the average time from the cycle start to the time the chart indicates signal after the process shift.

$$M = r'(I-Q)^{-1}\, t$$

t is the vector of the sampling intervals. Here, we set vectors r'=(1, 0, 0, 0, 0, 0, 0) and t' =(h, h, h, h, h, h, h).

T_3: Let t_0 be the average amount of time for searching each false alarm, and E(F) be the expected number of false alarms per cycle. Thus,

$$E(F) = r'(I-Q)^{-1}\, f$$

Where: $f' = (0,0,0,1,0,0,0)$. So the expected time for searching false alarm per cycle is t_0 E(F).

T_4: The time to identify and fix the assignable cause following a true alarm is a constant t_1.

As a result, The expected production cycle time of DSVSS, E(T), is:

$$E(T) = M + t_0 E(F) + t_1$$

If we define:

V_0= the hourly profit earned when the process is in control state.

V_1= the hourly profit earned when the process is in out-of-control state.

C_0= the average search cost if the given signal is false

C_1= the average cost to find the assignable cause and adjust the process to in-control state

S = the cost for each inspected item

Therefore, the expected net profit from a production cycle is:

$$E(C) = v_0(1/\lambda) + v_1(\mathrm{M} - 1/\lambda) - c_0 E(F) - c_1 - sE(N)$$

Where $E(N)$ is the average numbers of inspected items per cycle, and it is:

$$E(N) = r'(I-Q)^{-1}\, \eta$$

$$\bar{n} = n_1(p_{13} + p_{53}) + n_2(p_{23} + p_{63})$$

Where $\eta = (n_1\,, n_2, \bar{n} + n_3, n_1, n_1, n_2, \bar{n} + n_3)$.

At end, the expected loss per hour E(L) is:

$$E(L) = V_0 - \frac{E(C)}{E(T)}$$

The economic design model of DSVSS is presented as:

$$\text{Min } E(L)$$

s.t.

$$0 < w_1 \le w_2 < k_1 < k_u$$
$$0 < k_2 < k_1$$
$$0 < n_2 < n_1$$
$$n_1, n_2, n_3 \in N$$

Where the k_u is the maximum tolerance value of the control limit.

4. Solution Procedure

Genetic algorithms for optimization problems are used in many areas. so In this study, the GA is also selected to solve the optimal designs of DSVSS. E(L) is a function of the process parameters (t_0, t_1,λ, δ), the cost parameters (S, C_0, C_1, V_0, V_1), and the design parameters (n_1, n_2, n_3, h, w_1, w_2, k_1, k_2).We want to minimize the cost function by using GA. The GA contains the following major steps:

(1) Randomly generate primary solutions which have to satisfy the constraints.

(2) Compare fitness of solutions by objective function.

(3) Conduct crossover and mutation to get next generation solutions and then return to step (2) until a

convergent solution will be obtained.

5. A Numeric Example & Sensitivity Analysis

We set parameters such as v_0=500, v_1=250, c_0=250, c_1=500, s =5, t_0 =5, t_1=1, $\lambda = 0.02$, k_u =5, and δ=1.5 is considered for the economic design of DSVSS. The above mentioned values are substituted to construct the model, and then GA is used to find the optimal solution. By conducting repeated tests, we have found the best values to be set for the number of population, the crossover rate, and the mutation rate are 230, 0.8, and

0.25, respectively, and these settings of GA are applied for the solution of DSVSS. We obtain the optimal parameters of DSVSS n_1=10, n_2=12, n_3=6, h=4.65, k_1=3.25, k_2=4.5, w_1=2.05, w_2=2.38, and the expected E(L) value is 42.17.

The effects of model parameters on the optimal design of DSVSS are studied .Table1 shows the effect of model parameters on the optimal design of the DSVSS.

Table 1.

		n_1	n_2	n_3	h	k_1	k_2	w_1	w_2	E(L)
	400	8	9	5	5.05	2.97	3.7	1.8	2.5	34.07
v_0	500	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	600	9	10	4	3.71	2.8	3.5	1.71	2.1	49.08
	200	7	7	4	3.58	3.02	2.88	1.8	2.6	43.86
v_1	250	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	300	6	9	9	3.96	3.01	3.03	1.81	2.31	38.88
	200	7	8	6	3.81	3.04	3.1	1.94	2.23	41.23
c_0	250	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	300	7	9	6	3.68	3.05	3.08	2.01	2.45	41.39
	250	8	8	4	4.13	3.02	3.06	2.16	2.68	36.93
c_1	500	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	750	8	10	6	4.32	3.08	3.07	2.24	2.64	46.18
	0.01	6	10	9	4.47	3.14	3.06	1.82	2.40	25.93
λ	0.02	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	0.05	10	10	4	3.15	3.32	3.15	1.96	2.64	79.78
	1	14	15	12	4.68	2.86	2.65	1.82	2.25	50.27
δ	1.5	10	12	6	4.65	3.25	4.5	2.05	2.38	42.17
	2	6	6	3	3.64	3.43	4.25	2.16	2.48	37.16

The following conclusions can be derived:

(1) Increasing δ conspicuously increases the k_1, w_1 and w_2 but reduces the n_1, n_2, n_3 and h.

(2) Increasing λ conspicuously increases the n_1, k_1 but reduces n_3.

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